Combustion Response to Compositional Fluctuations

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This paper presents a mechanism by which the heterogeneity of composite solid propellants can contribute to combustion oscillations. The mechanism is based upon compositional fluctuations at frequencies evoked by characteristic dimensions of the fine structure and the mean burning rate. The combustion response to compositional fluctuations is derived using an appropriate composite propellant combustion model, and its properties are discussed in terms of the calculated results obtained. Of particular interest is the "concentration exponent," a parameter analogous to the more familiar pressure exponent of the burning rate but with respect to ammonium perchlorate concentration. Theory and data show that this parameter has a tremendous range of variability and can attain extraordinary values. It is thereby inferred that the compositional response can be a dominating factor in driving combustion instability and in explaining the effects of particle size on combustion response. Calculated results infer that it is a necessary ingredient of a strong response at lower frequencies. Although certain conclusions can be drawn about the compositional response itself, there are many questions that need to be answered before definite conclusions can be made about its relationship to combustion instability.

Nomenclature

A = parameter defined by Eq. (28)
a_1,a_2,a_3 = constants in the relation for perturbed surface temperature, Eq. (12)
B_1,B_2 = constants in the relation for the compositional response, Eq. (15)
b_1,b_2,b_3 = constants in the relation for perturbed surface temperature gradient, Eq. (10)
C_1,C_2 = constants in the relation for the pressure response, Eq. (14)
C_3,C_4 = other constants in the response relations
C_I = constant of integration for perturbed surface temperature, Eq. (7)
D = ammonium perchlorate (AP) particle size
d_j = characteristic dimension for the jth component of heterogeneity
E_w = activation energy of surface decomposition
f_j = frequency associated with the jth component of heterogeneity, Eq. (29)
H_f = binder heat of decomposition
H_L = AP heat of decomposition
i = $\sqrt{-1}$
K_ρ = dependence of propellant density on AP concentration, Eq. (9)
m = mass flux
n_p = pressure exponent, Eq. (18)
n_α = concentration exponent, Eq. (19)
p = pressure
R = universal gas constant
R_c = classical pressure-coupled response function, Eq. (24)
R_p = overall pressure-coupled response function, Eq. (30)
R_α = compositional response function, Eq. (25)
r = burning rate
T_i = initial bulk temperature of solid propellant
T_w = surface temperature of solid propellant
t = time
y = dimensionless distance, normalized by solid-propellant characteristic thermal wave thickness
α = AP concentration
θ = dimensionless temperature $(T-T_i)/(T_{w_0}-T_i)$
λ = complex quantity characteristic of perturbation solution
ρ = propellant density
ρ_f = binder density
ρ_{ox} = AP density
σ_p = burn rate/temperature sensitivity, Eq. (22)
σ_T = dependence of propellant surface temperature on conditioning temperature, Eq. (21)
Ω = dimensionless frequency, normalized by characteristic time for solid-propellant thermal wave

Subscripts and Superscripts

0 = mean value
$(\ )'$ = perturbation

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Introduction

Ammonium perchlorate (AP) particle size distribution has a major effect on the tendency of a composite solid propellant to drive combustion instability.[1,2] This effect has been observed in laboratory combustion response data and in the course of rocket motor development programs (in particular, see the references cited in Ref. 2). The examples cover a broad range of oscillatory frequencies, including encounters with longitudinal and tangential mode instabilities.

Classical theories of the combustion response[3] do not specifically address AP particle size and are incapable of describing or explaining the features of the experimental data that appear to depend upon particle size. For example, the theories predict a single resonant frequency, whereas recent data can be interpreted to show multiple peaks in the frequency dependence.[4-7] These data have been acquired in laboratory devices that afford better frequency resolution than T-burners and measurement of the phase as well as the magnitude of the response. Another example is that the theories predict the effects in terms of steady-state ballistics properties, whereas some experience has shown the differences to be due to particle size changes per se (see Ref. 2 citations).

As a general trend, finer sizes tend to drive higher frequencies, but the mechanism has not been ascertained. More recent theories do address particles size[2] but, with one exception, are expressed in terms of the particle size effects on the ballistics properties, so they are really extensions of the classical theory. The exception is the work of Lengelle and Williams,[8] who applied a perferred frequency concept based upon the mean burning rate and the fine structure dimensions of the composite propellant heterogeneity. This paper discusses a novel aspect of that concept, by which the particle size distribution can contribute to the combustion response.

Most combustion response analyses have treated the combustion response with respect to pressure perturbations only. Several have treated velocity perturbations in the context of "velocity coupling."[9-11] One interesting work has treated perturbations in the radiation imposed upon the propellant[12] and another has recently considered combustion efficiency perturbations.[13] The unique feature of this paper is its treatment of combustion response with respect to compositional fluctuations, i.e., perturbations in the AP concentration. Assuming that such fluctuations are relevant to combustion instability, it would be but one aspect of that problem. This paper's scope is limited to that aspect. Except for some heuristic remarks, it does not go further into the questions of coupling to the acoustic (pressure and velocity) waves, coherence of the response over the burning surface, or frequency distributions of the heterogeneities of the actual propellants. These are subjects for additional work. The response to compositional fluctuations is itself of interest.

Existence of Compositional Fluctuations

The one-dimensional, steady-state burning of composite propellants is, in reality, a statistical average of three-dimensional, unsteady burning on a local microscopic scale. If a plane is passed through a composite propellant, local fluctuations in the propellant composition will be encountered. On the smallest scale, the variation is from total AP (a particle) to total binder (the space between particles). On a slightly larger scale, considering an individual particle and the portion of the binder associated with it in various stages of burning, the local oxidizer weight fraction can vary by a factor of about four. Several steady-state burning models are based upon an averaging of such variations.[14]

Typical composite propellants contain of the order 10^7-10^{10} particles per cubic centimenter, depending upon the size distribution. In a well-mixed propellant, the particles may be settled in uniform arrays that appear repetitively. This is easy to visualize for the ideal case of a packed bed of unimodal spheres. Periodic, although nonsinusoidal, fluctuations in the microscopic particle concentration may be computed readily for a plane that is moving through this ideal geometry. Therefore, it is hypothesized that macroscopic periodicities may exist in real propellants—although in a much more complicated way because of the distribution of particle sizes and nonideal shapes. For the ideal case, the magnitude of the fluctuations is within a factor of 1.3 (down from the factor of 4 for the localized fluctuations); a Fourier analysis can produce several frequency components (depending upon the planar orientation) for the single particle size. On this basis, fluctuations in real propellants would be expected to be within a few percent (i.e., much weaker) and with broadband frequency content. Experimental evidence of the compositional fluctuations has appeared in various forms in the literature.[15-20] However, there is not agreement that the propellant microstructure is an inherent source of such fluctuations in practical propellants. Another basis for compositional fluctuations during combustion has been proposed by Price.[21]

Fluctuations in the propellant formulation give rise to fluctuations in various combustion parameters, which in turn affect the burning rate. Thus, there is a plausible cause-and-effect relationship between the compositional and burning rate fluctuations at constant pressure, with interesting second-order feedback possibilities. The scope of combustion parameters that will fluctuate depends upon the particular combustion model used, but the most important involve the heat feedback from the flame. In particular, the heat release in the flame can acquire nonlinearities (large, asymmetric variations) even for linearized compositional fluctuations. The present analysis will, however, be restricted to the linearized problem. A "computational response function" can be defined as

$$R_\alpha = \left(\frac{m'/m_0}{\alpha'/\alpha_0}\right)_{p,T_i} \tag{1}$$

for a constant pressure and conditioning temperature.

Analysis of the Compositional Response

Employing the usual set of assumptions, the mathematical analysis is a straightforward extension of the familiar derivations for the linear pressure-coupled response function.[3] The perturbed transport of energy in the solid propellant can be expressed as

$$\frac{d^2\theta'}{dy^2} - \frac{d\theta'}{dy} - i\Omega\theta' = 0 \tag{2}$$

One boundary condition is defined by the absence of perturbations at a large depth in the solid,

$$\theta'(-\infty) = 0 \tag{3}$$

The second boundary condition is defined by the perturbed heat flux at the surface of the solid,

$$\frac{d\theta'(0^-)}{dy} = \frac{d\theta'(0^+)}{dy} - \frac{m'}{m_0}[\alpha_0(H_L - H_f) + H_f] - \frac{\alpha'}{\alpha_0}[\alpha_0(H_L - H_f)] \tag{4}$$

Note that the heterogeneity arises through perturbations in α and the Beckstead-Derr-Price (BDP) model[22] is used to give content to the heat of gasification in terms of the oxidizer and binder contributions.

Conditions at the instantaneous oscillating surface ($y = 0^-$) are related to conditions at the mean surface ($y = 0$) by

$$\theta'(0^-) = \theta'(0) + \frac{i}{\Omega}\left(\frac{m'}{m_0} - \frac{\rho'}{\rho_0}\right) \equiv \theta'_w \tag{5}$$

$$\frac{d\theta'(0^-)}{dy} = \frac{d\theta'(0)}{dy} + \frac{i}{\Omega}\left(\frac{m'}{m_0} - \frac{\rho'}{\rho_0}\right) \equiv \frac{d\theta'_w}{dy} \tag{6}$$

where

$$\theta'(0) = C_1 \tag{7}$$

and

$$\frac{d\theta'(0)}{dy} = \lambda C_1 \tag{8}$$

The density perturbation terms in Eqs. (5) and (6) are new and arise from the heterogeneity,

$$\frac{\rho'}{\rho_0} = \frac{(\rho_{ox}/\rho_f) - 1}{(\rho_{ox}/\rho_f)[(1-\alpha_0)/\alpha_0] + 1}\,\frac{\alpha'}{\alpha_0} \equiv K_\rho \frac{\alpha'}{\alpha_o} \tag{9}$$

The tedious part of the perturbation analysis is in the derivation of $d\theta'(0^+)/dy$. Using the BDP model, this term is a function of perturbations in the propellant surface structure, flame heights, competing flame parameter, and diffusion flame heat release.[23] The perturbations in these various parameters can be expressed in terms of perturbations in surface temperature, pressure, and oxidizer concentration through the various relationships in the BDP model as

$$\frac{d\theta'(0^+)}{dy} = b_1\theta'_w + b_2\frac{\alpha'}{\alpha_0} + b_3\frac{p'}{p_0} \tag{10}$$

The solution may then be expressed as

$$\theta'_w(\lambda - b_1) - \frac{K_\rho}{\lambda}\frac{\alpha'}{\alpha_0} = -\frac{m'}{m_0}\left[\alpha_0(H_L - H_f) + H_f + \frac{1}{\lambda}\right] + \frac{\alpha'}{\alpha_0}[b_2 - \alpha_0(H_L - H_f)] + b_3\frac{p'}{p_0} \tag{11}$$

A relation for θ'_w is also available from the BDP model in terms of perturbations in the mass flux, pressure, and oxidizer concentration as

$$\theta'_w = a_1\frac{m'}{m_0} + a_2\frac{\alpha'}{\alpha_0} + a_3\frac{p'}{p_0} \tag{12}$$

Substitution for θ'_w into Eq. (11) yields

$$\frac{m'}{m_0}\left[a_1(\lambda - b_1) + \alpha_0(H_L - H_f) + H_f + \frac{1}{\lambda}\right] = \frac{\alpha'}{\alpha_0}\left[b_2 - \alpha_0(H_L - H_f) - a_2(\lambda - b_1) + \frac{K_\rho}{\lambda}\right] + \frac{p'}{p_0}[b_3 - a_3(\lambda - b_1)] \tag{13}$$

For pressure oscillations only, the classical response function takes the familiar form of

$$R_c = \frac{C_1\lambda + C_2}{C_3 + (1/\lambda) + C_4\lambda} \tag{14}$$

For oscillations in the oxidizer concentration at constant pressure, the compositional response function takes the form

$$R_\alpha = \frac{B_1\lambda + B_2 + (K_\rho/\lambda)}{C_3 + (1/\lambda) + C_4\lambda} \tag{15}$$

It is interesting that the heterogeneity component is in a form similar to the classical component. The differences are the parametric constants in the numerator (B_1 and B_2 rather than C_1 and C_2) and an additional term representing the density perturbation (K_ρ/λ). One of the constants in each expression can be replaced by a characteristic exponent representing the respective value at zero frequency (where $\lambda = 1$).

Table 1 Ranges of combustion parameters

n_p	0.2-0.8
$\sigma_p(T_{w_0} - T_i)$	0.3-1.5
$\sigma_T/[\sigma_p(T_{w_0} - T_i)]$	0.03-0.1
n_α	3-60
K_ρ	0.6-0.9

Eliminating C_2 and B_2, there results

$$R_c = \left[n_p + \frac{C_1(\lambda - 1)}{1 + C_3 + C_4}\right] \div \left[1 - \frac{1}{1 + C_3 + C_4}\left(1 - \frac{1}{\lambda}\right) + \frac{C_4(\lambda - 1)}{1 + C_3 + C_4}\right] \tag{16}$$

$$R_\alpha = \left[n_\alpha + \frac{B_1(\lambda - 1)}{1 + C_3 + C_4} - \frac{K_\rho}{1 + C_3 + C_4}\left(1 - \frac{1}{\lambda}\right)\right] \div \left[1 - \frac{1}{1 + C_3 + C_4}\left(1 - \frac{1}{\lambda}\right) + \frac{C_4(\lambda - 1)}{1 + C_3 + C_4}\right] \tag{17}$$

where

$$n_p \equiv \left(\frac{\partial \ln m}{\partial \ln p}\right)_{\alpha, T_i} \quad \text{pressure exponent} \tag{18}$$

and

$$n_\alpha \equiv \left(\frac{\partial \ln m}{\partial \ln \alpha}\right)_{p, T_i} \quad \text{concentration exponent} \tag{19}$$

Equations (16) and (17) are in the form that results from application of the Zel'dovich-Novozhilov method,[24] whereby the various constants are given meaning in terms of key steady-state combustion parameters. It turns out that the terms involving C_1 and B_2 are negligible. The remaining constants become

$$\frac{1}{1 + C_3 + C_4} = \sigma_p(T_{w_0} - T_i) \tag{20}$$

$$\frac{C_4}{1 + C_3 + C_4} = \left(\frac{\partial T_w}{\partial T_i}\right)_{p,\alpha} \equiv \sigma_T \tag{21}$$

where

$$\sigma_p \equiv \left(\frac{\partial \ln m}{\partial T_i}\right)_{p,\alpha} \quad \text{temperature sensitivity} \tag{22}$$

and

$$K_\rho \equiv \left(\frac{\partial \ln \rho}{\partial \ln \alpha}\right)_{p, T_i} \tag{23}$$

The final expressions then become

$$R_c = \frac{n_p}{1 - \sigma_p(T_{w_0} - T_i)[1 - (1/\lambda)] + \sigma_T(\lambda - 1)} \tag{24}$$

$$R_\alpha = \frac{n_\alpha - K_\rho\sigma_p(T_{w_0} - T_i)[1 - (1/\lambda)]}{1 - \sigma_p(T_{w_0} - T_i)[1 - (1/\lambda)] + \sigma_T(\lambda - 1)} \tag{25}$$

Parametric Studies

Range of Variables

The combustion parameters comprising the response functions are listed in Table 1, together with the ranges in the values of interest. Except for the parameters involving the surface temperature, they can be measured with sufficient confidence that a theory is not required to calculate them.

Surface temperature presents a problem because a composite propellant does not have a single surface temperature[14] and because there is not complete agreement on the range of surface temperatures in the binder.[25] Attempted measurements and deductions regarding the binder are not fully accepted. The situation is much better for the AP, which is the major propellant ingredient—it is generally agreed that the AP surface temperature is in the area of 850-900 K. The Cohen-Strand model[26] uses input data reflecting the view that the polybutadiene (HTPB) binder is generally at a higher surface temperature than AP. This model was used to calculate the surface temperatures used in generating the values in Table 1. It is assumed that a weighted average of ingredient surface temperatures can be used to represent "the surface temperature" in calculating the response functions. Whereas something is known about the surface temperature, virtually nothing is known about its dependence upon T_i. The model was also used to calculate the values and ranges for this parameter.

It turns out that the parameters σ_p, σ_T, and $T_{w_0}-T_i$ are not mutually exclusive. Combustion model calculations reveal correlations between them to the extent that the ranges of the Table 1 parameters need not reflect the independent ranges of these components. A very simple combustion model can be used to illustrate the relationships. Combining an Arrhenius decomposition law with the solid-phase heating requirement, assuming no heat of decomposition and a gas-phase heating of the solid independent of T_i, there results

$$\sigma_p(T_{w_0}-T_i)=A/(A+1) \tag{26}$$

$$\sigma_T/[\sigma_p(T_{w_0}-T_i)]=1/A \tag{27}$$

where

$$A=E_w(T_{w_0}-T_i)/(RT_{w_0}^2) \tag{28}$$

A typical value for A is 8, which does not vary significantly for the calculated range of surface temperatures. Use of the Cohen-Strand model and experimental as well as calculated ranges of σ_p provide a greater range to the parameters than would be inferred from Eqs. (26-28). However, it is a far smaller range than would result in the absence of these relationships.

The range of pressure exponents is typical of composite propellants. Lower (even negative) values would be desirable for most applications, but are difficult to achieve. Higher values are sometimes encountered in special formulations or certain pressure regimes, but would generally render the propellant impractical. In any event, Eq. (24) shows that R_c is proportional to n_p and could instead be written as R_c/n_p (as is often done in the literature).

The parameter n_α, which includes K_ρ if determined from linear burning rates, appears as a new parameter in the context of combustion theory. It has not per se been treated in the literature, although information exists from which determinations can be made. A set of theoretical results acquired with the Cohen-Strand model is presented in Table 2 and some experimental data[27,28] in Table 3. The magnitudes and variabilities in this parameter are striking. Values are at least much higher, and range up to two orders of magnitude greater, than the pressure exponent. They are strongly dependent upon the AP concentration, particle size distribution, and pressure. One set of theoretical results may be compared with King's data. It is observed that the pressure effect is predicted correctly, but the predicted values are less than the experimental values. Miller's data suggest that narrowing the size distribution reduces n_α. If this is correct, then a truly unimodal size (as in Table 2) will provide the minimum n_α for a given mean size. An intriguing result is the high negative values predicted on the oxidizer-rich side of stoichiometry (not included in Table 1), where the flame temperature decreases with increasing AP concentration.

Given the large magnitudes of n_α, the term involving K_ρ in Eq. (25) can generally be neglected. Thus, the behavior of R_α/n_α would closely follow the behavior of R_c/n_p. But, since n_α is so much larger than n_p, it can be asserted that a mechanism based upon compositional variations is going to have a potentially dominating influence on the unsteady combustion.

Table 2 Calculated values of the concentration exponent[a] using the Cohen-Strand model

Pressure, MPa	D, μm 0.7	5	20	90	200	400
			$\alpha_0=0.73$ ($K_\rho=0.618$)			
1.4	8.1	8.2	8.3	4.1	3.6	3.8
3.4	8.4	8.6	5.4	2.8	3.7	4.6
6.8	8.9	9.4	4.4	3.0	4.7	6.0
13.6	9.3	5.8	3.1	4.8	6.2	7.2
			$\alpha_0=0.80$ ($K_\rho=0.719$)			
1.4	5.4	5.6	6.2	3.9	2.9	2.6
3.4	5.6	6.2	7.1	3.2	2.9	3.2
6.8	5.7	6.8	4.2	3.0	3.5	4.8
13.6	6.0	7.5	3.4	3.6	5.0	6.9
			$\alpha_0=0.88$ ($K_\rho=0.853$)			
1.4	1.6	2.4	4.7	13.0	6.5	4.6
3.4	1.9	4.0	9.9	7.0	4.9	4.2
6.8	2.2	5.3	14.0	3.4	4.3	4.7
13.6	3.0	10.3	7.3	4.3	5.1	7.1
			$\alpha_0=0.92$ ($K_\rho=0.928$)			
1.4	−6.5	−23.1	−62.5	−23.9	−13.4	−10.6
3.4	−10.8	−47.8	−45.1	−16.3	−11.8	−14.8
6.8	−18.2	−71.9	−28.1	−12.1	−15.6	−30.0
13.6	−31.9	−45.2	−17.3	−16.6	−32.3	−59.4

[a] $n_\alpha=\left(\frac{\partial \ln r}{\partial \ln \alpha}\right)_{p,T_i}+K_\rho$.

Table 3 Experimental data for the concentration exponent

King, α = 0.73-0.77, 20 μm AP: p, MPa	n_α	Lockheed Propulsion Co. composite propellant, 6.8 MPa: α	n_α
1.4	11.2	0.78-0.80	6.3
3.4	10.1	0.80-0.82	8.6
6.8	8.9	0.82-0.84	10.4
13.6	6.2	0.84-0.86	7.8
		0.86-0.88	6.2

Miller trimodal propellants, $\Delta\alpha=0.02$

	D, μm					
Coarse	400	400	400	200	200	200
Medium	20	20	50	20	20	50
Fine	2	6	20	2	6	20
p, MPa	n_α					
2.0	26.0	25.3	12.3	20.5	22.2	5.4
3.4	36.3	32.1	12.4	15.7	29.3	11.2
6.8	61.0	37.0	11.6	20.9	31.1	8.6
13.6	40.1	25.9	12.6	25.3	22.0	3.7

Parametric Results

A plot of the real part of R_α/n_α vs dimensionless frequency Ω, with variable $\sigma_p(T_{w_0}-T_i)$, is shown in Fig. 1. These curves are for $K_\rho/n_\alpha=0$ and are therefore curves for R_c/n_p as well. The effect of $\sigma_p(T_{w_0}-T_i)$ is observed to be quite pronounced. Increases in this parameter substantially increase the peak response magnitude and sharpen the peak (or narrow the peak region). The value of 1.5 is close to an intrinsic instability limit. Therefore, large values of temperature sensitivity are undesirable. However, there is not much of an effect upon the peak response frequency, which occurs at an Ω of about 11.5 for these conditions.

Figure 2 shows the effect of varying the proportionality involving σ_T on R_α/n_α, for $K_\rho/n_\alpha=0.2$. The effect of R_c/n_p is

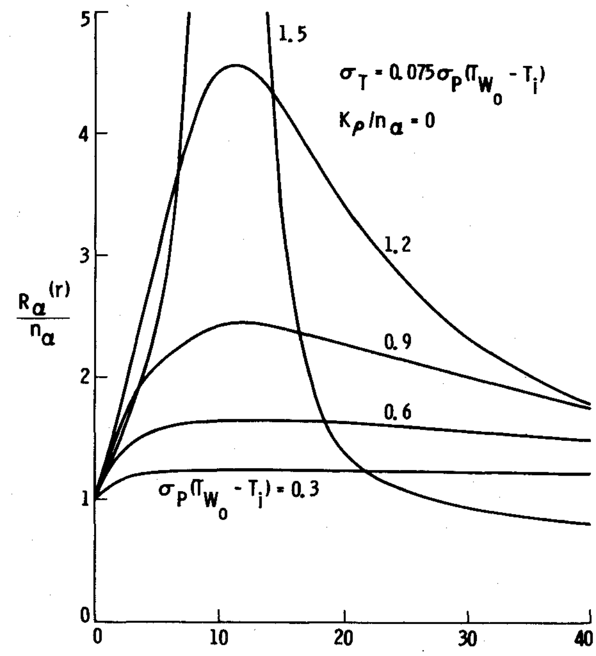

Fig. 1 Effect of temperature sensitivity of burn rate on the compositional response.

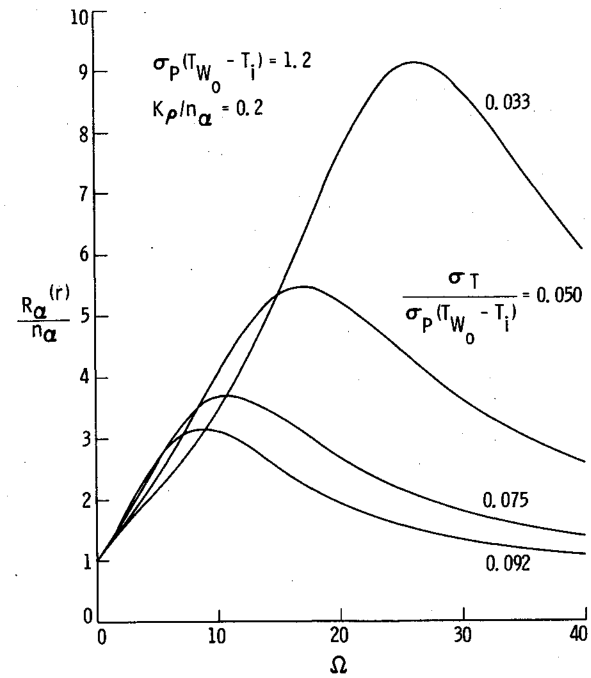

Fig. 2 Effect of temperature sensitivity of surface temperature on the compositional response.

qualitatively the same and would be identical for $K_\rho/n_\alpha=0$. Decreasing the proportionality tends to increase substantially the peak response magnitude and frequency and to broaden the peak region. Thus, a larger sensitivity of the surface temperature to the conditioning temperature would be desirable. However, it turns out that increases in σ_p tend to go with increases in σ_T, so there is a tradeoff that limits the extremes of response function behavior. Ideally, one would want a high sensitivity of the surface temperature together with a low sensitivity of the burn rate. That appears to be a contradiction, but combustion models can be explored for possible ways to optimize the combination.

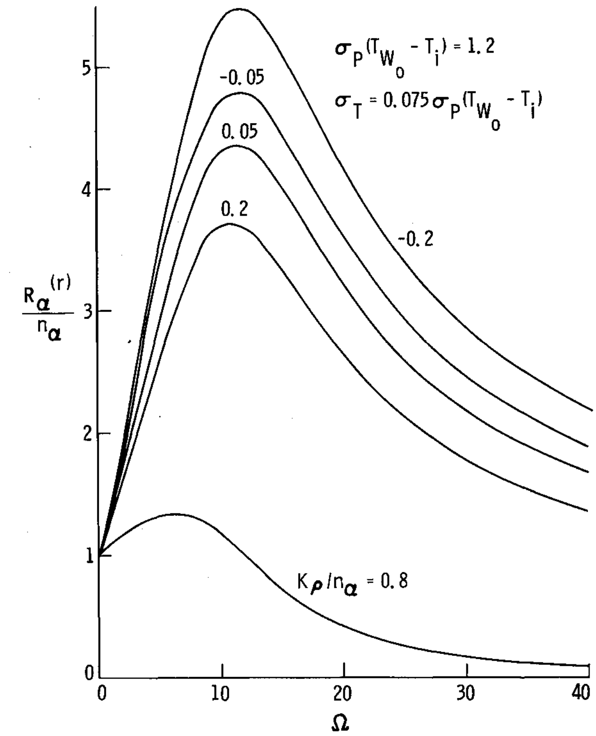

Fig. 3 Effect of density parameter on the compositional response.

Figure 3 shows the effect of varying K_ρ/n_α on R_α/n_α. Making K_ρ/n_α less positive (or more negative) tends to increase the peak response magnitude and frequency, but the effect becomes comparatively small at small absolute values of K_ρ/n_α. The peak response region broadens as the peak value increases. Some interesting behavior is inferred at AP concentrations traversing stoichiometry, which occurs at about 90% AP with an HTPB binder. The values of K_ρ/n_α will gyrate from low to high positive values, go through a sign change, and then gyrate from high to low negative values. Thus, R_α/n_α will be quite sensitive to α in this region.

Effects of Propellant Formulation and Pressure

The effects of propellant formulation and pressure upon these governing combustion parameters can be calculated from combustion models in the absence of data. The problem is that different models may give different results, so this aspect of the parametric study must be viewed cautiously. Using the Cohen-Strand model, the results are summarized as follows. For the AP concentrations of interest, in the range of 80-88%, the product $\sigma_p(T_{w_0}-T_i)$ tends to increase with increasing particle size and pressure and to decrease with increasing AP content. The σ_T proportionality also decreases with increasing AP content. It tends to go through a maximum at an intermediate particle size that increases with increasing pressure. The pressure effect is dependent upon the particle size. For fine sizes, the proportionality goes through a minimum at intermediate pressures. For intermediate sizes, it goes through a maximum at intermediate pressures. For coarse sizes, it increases with increasing pressure. The largest values occur at combinations of high pressure and intermediate-coarse size AP, which help to countermand the largest values of $\sigma_p(T_{w_0}-T_i)$. The predicted effects upon n_α were shown in Table 2 and are more difficult to summarize.

Calculated peak values for the classical pressure-coupled response function are shown in Table 4 for comparison with the calculated peak values for the compositional response function given in Table 5. It is observed that the computational response is always much larger, sometimes by an order of magnitude. The effects of particle size, AP concentration,

Table 4 Calculated peak values of the classical pressure-coupled response function

	D, μm					
Pressure, MPa	0.7	5	20	90	200	400
		$\alpha_0 = 0.80$				
1.4	1.5	1.3	0.86	0.83	0.88	1.0
3.4	1.5	1.1	0.38	2.1	2.3	2.7
6.8	1.4	0.89	1.8	3.3	2.9	2.1
13.6	1.3	0.39	5.0	3.1	2.3	0.82
		$\alpha_0 = 0.88$				
1.4	1.3	1.2	1.0	0.37	0.69	0.86
3.4	1.3	1.1	0.67	1.3	1.9	2.4
6.8	1.3	1.0	0.31	2.6	3.3	3.7
13.6	1.3	0.67	1.7	3.6	3.6	3.3

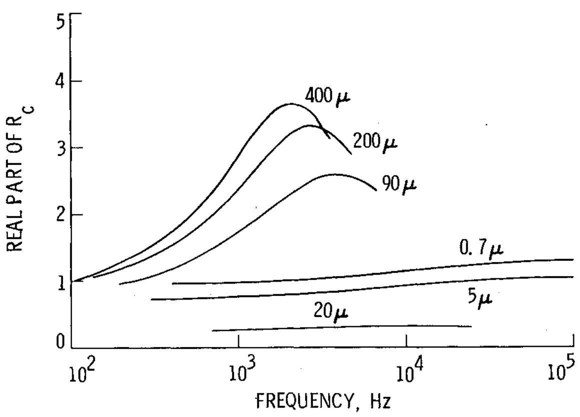

Fig. 4 Effect of AP particle size on the R_c combustion response (88% AP, HTPB at 6.8 MPa).

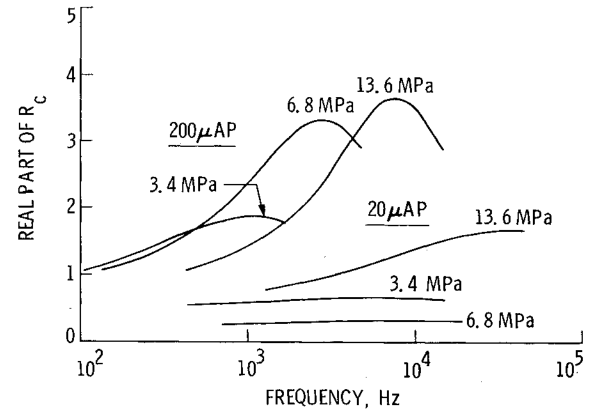

Fig. 5 Effect of pressure on the R_c combustion response for two AP sizes (88% AP, HTPB).

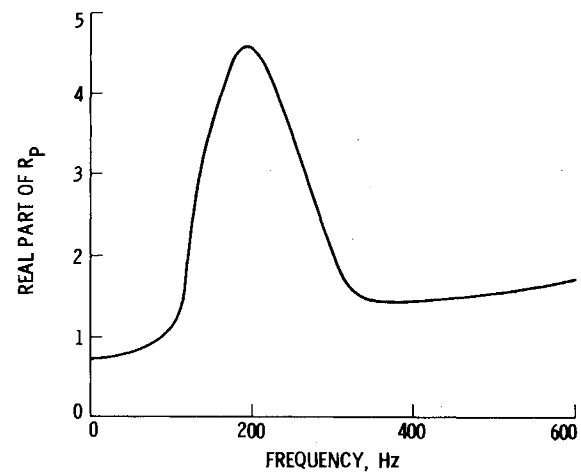

Fig. 6 Calculated response function curve for an assumed heterogeneity form function (88% of 90 μm AP, HTPB at 6.8 MPa).

and pressure are not systematic, reflecting a complicated interaction between the behavior of the surface temperature parameters and the exponents. The largest peak values tend to occur at combinations of high pressure and intermediate-coarse particle size. The lowest peak values occur at particle sizes that decrease with increasing pressure. These are boxed for illustration and come about largely because of the lower exponent.

The peak response magnitude is not of itself an indication of relative stability. Frequency is also important. The peak response frequencies are dominated by trends in the burning rate. These frequencies increase with increasing pressure and decreasing particle size, as illustrated by Table 6. The exceptions come about because of the K_p term in Eq. (25); this term also causes the peak response frequencies of R_c and R_α to differ somewhat.

Figures 4 and 5 illustrate the effects of AP particle size and pressure on the real part of R_c vs frequency. Results for R_α are similar, except that the ordinate scale would need to be increased by a factor of five to accommodate the larger values calculated under these conditions. There is no intent to provide an exhaustive study at this point because the results are strongly colored by the combustion model, so there is danger in making too much of them. The main point of the illustration is that there are significant effects of particle size and pressure, this is what they tend to look like, and the qualitative trends are interdependent (sometimes they are systematic, sometimes not, which is consistent with the confusing nature of the problem gleaned from experience). Some specific results of interest are that the response functions tend to be largest with coarse particles at frequencies of the order 10^3 Hz and that the response functions with fine particles tend to be larger than with coarse particles at frequencies of the order 10^4 Hz. Since the high frequencies are normally associated with tangential modes (and tangential modes do not have to overcome nozzle damping), the finer sizes are capable of driving even though the response is relatively low. At low frequencies, it is apparent that some additional mechanism is required to achieve the large values and peak values of the response function that are measured experimentally at frequencies of the order 10^2 Hz. This additional mechanism would also be operative at high frequencies.

Viewing these results in terms of the combustion model mechanisms, it appears that a kinetically limited flame produces a stronger instability or sensitivity than a diffusion-limited flame and/or that the tendency toward instability varies inversely with the energy release in the controlling flame. In a gross sense, the effects of particle size and pressure can be related to the regimes of diffusion flame control and AP flame control.

Relationship to Combustion Instability

It is hypothesized that a heterogeneity contribution to combustion instability arises from periodicities in the composite propellant fine structure. Resulting perturbations in α will contribute to the acoustic driving if they occur at the same frequency as the acoustic waves. The frequencies of the perturbations are assumed to be evoked by the mean burning rate and the characteristic dimensions of the fine structure

$$f_j = r_0 / d_j \tag{29}$$

Considering the range of burning rates and possible characteristic dimensions in typical propellants, possible frequencies range on the order of 10-10^5 Hz. These correspond to the frequencies of interest for various forms of unstable burn-

Table 5 Calculated peak values of the compositional response function

Pressure, MPa	0.7	5	20	90	200	400
			D, μm			
			$\alpha_0 = 0.80$			
1.4	7.7	7.9	8.8	6.5	5.0	5.4
3.4	8.4	9.3	10.6	7.9	7.6	10.9
6.8	8.7	11.6	9.9	9.9	11.8	15.9
13.6	9.2	12.1	17.1	12.3	18.4	18.3
			$\alpha_0 = 0.88$			
1.4	1.8	2.9	6.1	18.0	10.8	8.2
3.4	2.2	4.9	13.8	14.9	11.8	11.4
6.8	2.7	7.4	19.7	9.6	14.6	18.2
13.6	3.9	14.9	17.9	15.5	19.5	29.6

Table 6 Calculated peak response frequencies (kHz) for the compositional response, $\alpha_0 = 0.8$

Pressure, MPa	0.7	5	20	90	200	400
			D, μm			
1.4	0.44	0.60	0.60	0.16	0.11	0.09
3.4	4.13	6.40	1.39	0.48	0.37	0.32
6.8	21.1	17.4 [a]	2.50	1.35	1.09	0.74
13.6	95.2	29.4	12.50	3.88	2.44	1.23

[a]Values in excess of 10 probably violate the quasisteady gas assumption.

ing. Combinations of a low burn rate and coarse dimensions will tend to favor low frequencies; combinations of a high burn rate and fine dimensions will tend to favor high frequencies.

Assuming a coupling between the pressure and compositional oscillations at a given frequency, the response function can be expressed more generally in terms of a total derivative as

$$R_p = R_c + R_\alpha \frac{\alpha'/\alpha_0}{p'/p_0} \tag{30}$$

Calculation of this response function requires knowledge of the amplitude and phase relationships between the pressure and compositional perturbations and, therefore, an analysis of the coupling mechanism or measurements of the perturbations. It is implied that the response will be device dependent and nonlinear in nature, which makes it even more difficult to achieve generalizations.

For purposes of illustration, Eq. (30) has been applied to a propellant consisting of 88% AP with a finite-width monomodal particle size distribution having a weight-mean diameter of 90 μm. The following heuristic assumptions were employed: α' and p' are in phase; for each jth particle size interval, the amplitude ratio [R_α multiplier in Eq. (30)] is given by the weight fraction of the particles in that interval, and the characteristic frequency is given by the mean size in that interval according to Eq. (29). The result is shown in Fig. 6. Unlike Fig. 4, there is now a strong peak response at a frequency on the order of 10^2 Hz. This is entirely due to the heterogeneity contribution as implemented by Eq. (30) and reflects the particle size distribution in accordance with the assumption. The fact that R_α is much larger than R_c is the key to this result. This effect of the heterogeneity would be expected to shift to higher frequencies for finer AP or a higher burn rate and to lower frequencies for coarser AP or a lower burn rate. Based upon the combustion model results, this effect would also be expected to be stronger with coarser AP because that is where the largest magnitude differences between R_α and R_c arise (Tables 4 and 5). A second peak will arise due to R_c, consistent with Fig. 4, and multiple peaks will arise if there is a discrete multimodality in the distribution of characteristic dimensions. Furthermore, it is possible to change the response function by changing the size distribution, even though the ballistics properties remain constant.

Conclusions

The combustion response to compositional fluctuations has been derived and characterized. Its properties are very similar to the classical combustion response to pressure perturbations, except that its magnitudes are much larger. The concentration exponent, or dependence of the burn rate on the AP concentration, appears as a significant new combustion parameter. It is an interesting parameter because of its tremendous range of variability. Parameters having a strong effect upon the response functions are the exponents and parameters comprised of burn rate/temperature sensitivity and surface temperature. Theoretical effects of propellant formulation and pressure upon the response functions are not systematic enough to make complete generalizations. However, the largest peak values tend to occur with coarser particles at higher pressures. Finer sizes tend to produce larger values at higher frequencies, but with smaller peaks.

A mechanism for the effect of compositional heterogeneities on the pressure-coupled combustion response function has been proposed. However, the mechanism is as yet unsubstantiated and much theoretical and experimental work remains to be done for its evaluation.

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